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Optimal Electron Beam Transport in TAU FEL wiggler

1. Introduction

The main purpose of this work was to get optimal electron beam transport parameters in TAU FELusing analytical calculations and numerical computer simlulations.
Computer programs used were: "ELOP" and "QuadOpt".

1.1. "ELOP".

"ELOP" is a program for computer simulation of electron dynamics in different magnetic and electric elements, such as magnetic coils, permanent magnets and magnets pairs, wiggler (which consists of permanent magnets), constant magnetic and electric fields, quadrupole magnets and so on. The main concept of the program is calculation of electron trajectory in given electric and magnetic field as the result of Lorentz force:
(see Y.Pinhasi, Doctorate Thesis)

$$\begin{aligned}
\frac{dv_x}{dz} &= \frac{1}{\gamma}\left\{-\frac{e}{m}\frac{1}{v_z}\left[E_x + (v_yB_z - v_zB_y) - v_x\frac{d\gamma}{dz}\right]\right\} \\
\frac{dv_y}{dz} &= \frac{1}{\gamma}\left\{-\frac{e}{m}\frac{1}{v_z}\left[E_y + (-v_xB_z + v_zB_x) - v_y\frac{d\gamma}{dz}\right]\right\} \qquad (1) \\
\frac{dv_z}{dz} &= \frac{1}{\gamma}\left\{-\frac{e}{m}\frac{1}{v_z}\left[E_z + (v_xB_y - v_yB_x) - v_z\frac{d\gamma}{dz}\right]\right\}
\end{aligned}$$

- here vx,vy,vz - electron velocity components, e,m -charge and mass of electron, Ex,Ey,Ez , Bx,By,Bz - electric and magnetic field components,
$\gamma = 1/\sqrt{1-v^2/c^2}$ - relativistic Lorenz factor.

1.2. "QuadOpt".

"QuadOpt" is a program developed in MathCad (in cooperation with Dr. Yosi Pinhasi) which lets us find out the values of quadrupole magnet exitation currents which are required to transform an electron beam when we are given beam waist positions and radii at the entrance and exit of the optical system. The main idea of this procedure was to calculate ABCD matrix of an "optical" system, consisting of 4 quadrupoles (be cause 4 degrees of freedom are nececcary for this task) ,which is a product of matrices of the optical elements of the system. The electron beam was modeled by a gaussian beam with wavelength $\lambda=\varepsilon$, where ε is beam emittance.

We consider the quads system used to inject the acclerated electron beam into the wiggler (see Fig. 1). The focusing quadrupoles system consists of 4 quads

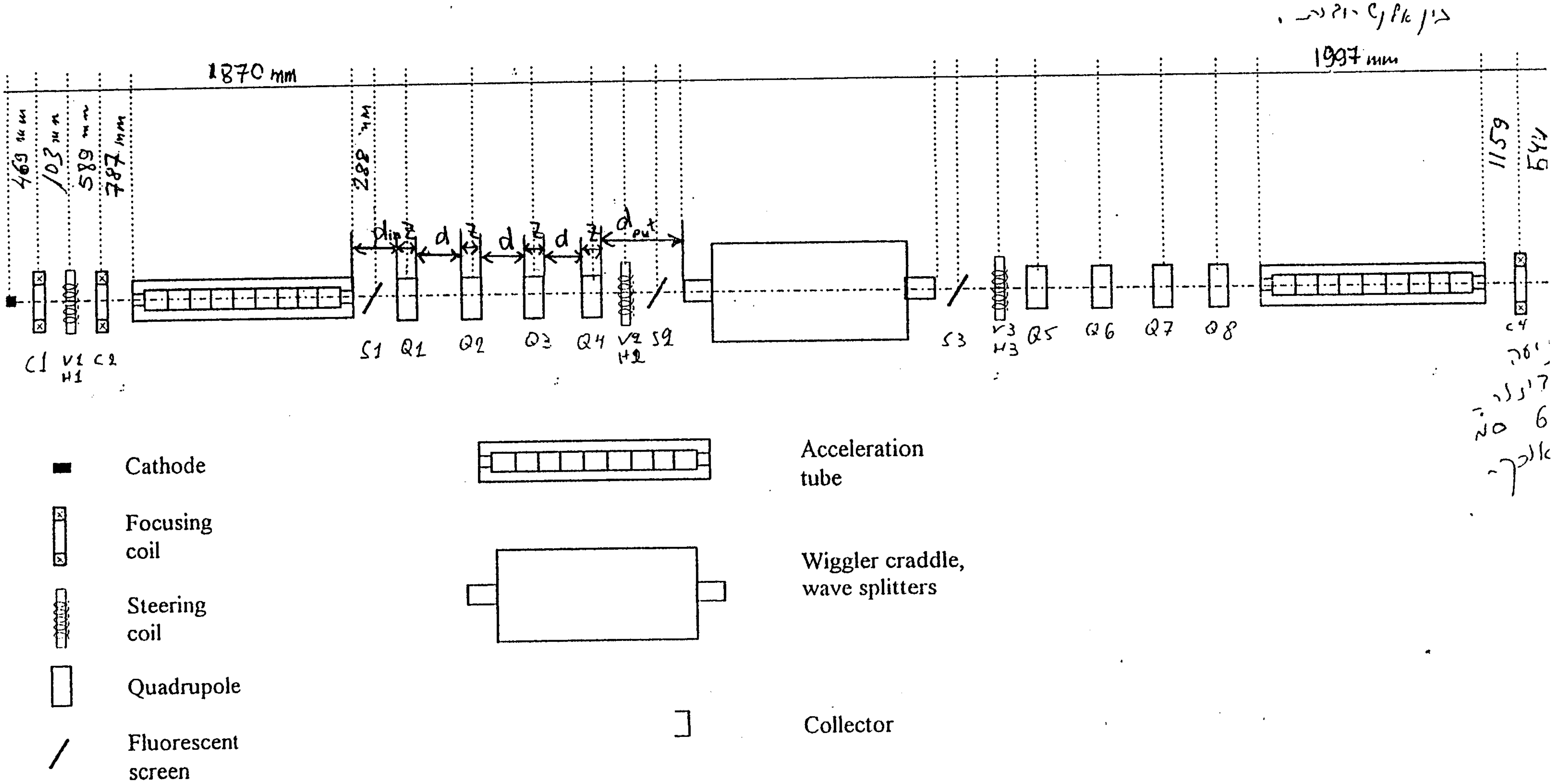


Fig. 1 Parameters used in "RandOpt"

separated with equal distance d. We assume that the waist is formed at the accelerating tube exit (distance d_{in} before the face of the first quad) and at the entrance to the wiggler (distance d_{out} often the face of the last quad). The quadrupole magnet is known to focus in one transverse direction and defocus in the other perpendicular direction, so to get focusing in two directions it is necessary to apply opposite currents in nearest quads. When treating a quadrupole magnet as an optical element, its "ABCD " matrix could be represented as (see M.Reiser, "Theory and design of charged particle beams")

$$Mc(I) = \begin{bmatrix} \cos(\sqrt{aI}z) & \frac{1}{\sqrt{aI}}\sin(\sqrt{aI}z) \\ -\sqrt{aI}\sin(\sqrt{aI}z) & \cos(\sqrt{aI}z) \end{bmatrix} \qquad (2)$$

for converging quadrupole and

$$Md(I) = \begin{bmatrix} \cosh(\sqrt{aI}z) & \frac{1}{\sqrt{aI}}\sinh(\sqrt{aI}z) \\ -\sqrt{aI}\sinh(\sqrt{aI}z) & \cosh(\sqrt{aI}z) \end{bmatrix} \qquad (3)$$

for diverging one, which can be obtained from (2) by changing I to - I.
In (2) and (3) a - is the quadrupole parameter, I - quad current, z quad width.

Designating free-space propagation matrix as

$$D(d) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \qquad (4)$$

we compose a full 4-quads x- and y-transport matrices as

$$Mx(I1,I2,I3,I4) = D(dout)\cdot Md(I4)\cdot D(d)\cdot Mc(I3)\cdot D(d)\cdot Md(I2)\cdot D(d)\cdot Mc(I1)\cdot D(din)$$
$$My(I1,I2,I3,I4) = D(dout)\cdot Mc(I4)\cdot D(d)\cdot Md(I3)\cdot D(d)\cdot Mc(I2)\cdot D(d)\cdot Md(I1)\cdot D(din) \qquad (5)$$

Then we use the well-known "ABCD-law" (see Yariv, "Optical Electronics") to calculate the electron gaussian beam q-parameter transformation:

$$qout = \frac{A\cdot qin + B}{C\cdot qin + D} \qquad (6)$$

and demand that qin and qout have the form of

$$q = i\cdot\pi\cdot\frac{w_0^{\,2}}{\varepsilon} \qquad (7)$$

or separating real and imaginary parts in (7),

$$Re(q) = 0 \qquad (7a)$$

$$Im(q) = \pi\cdot\frac{w_0^{\,2}}{\varepsilon} \qquad (7b)$$

which simply means being in the waist of the gaussian beam.

Transforming qin with matrices Mx and My and demanding that q_{out} satisfies

(7a) and (7b) we get a set of 4 equations for currents I1,I2,I3,I4. Solving thiese equations by iterations we get the set of the optimal currents which would focus the gaussian electron beam to the required parameters (waist position and radius) at the exit.

Note: the required parameters inserted in the program are: $W0_{x\ in}$, $W0_{y\ in}$, $W0_{x\ out}$, $W0_{y\ out}$. The solution is for the quad currents I1, I2, I3, I4. In principle one can expect for 4 solutions. When $W0_x = W0_y$ the solutions degenerate into two. However, we were not able so far to find more than one solution. With all the sets of initial conditions with which we started the iteration process.

2. Wiggler structure.

TAU FELWiggler consists of 26 periods of magnets in the Hallbach configuration, where each period consists of 4 permanent magnet pairs (see Fig.2) .There are also additional entrance and exit "correcting" magnets and "long" lateral focusing magnets which are needed to eliminate declination of the electron beam at the entrnce and exit and focus the beam along the wiggler.

The purpose of the additional entarnce correcting magnets is to provide angular shift (with the vertical half-magnets) and parallel displacement shift (with the axial antiparallel half-magnets). There is a symmetric configuration of additional correcting magnets in the wiggler exit. The purpose of the first (full-size) pair of additional magnets at the exit is to achieve full symmetry of the wiggler (see Fig. 2).

All parameters and dimensions of TAU FEL wiggler are summarized in Table 1 and Table 2.

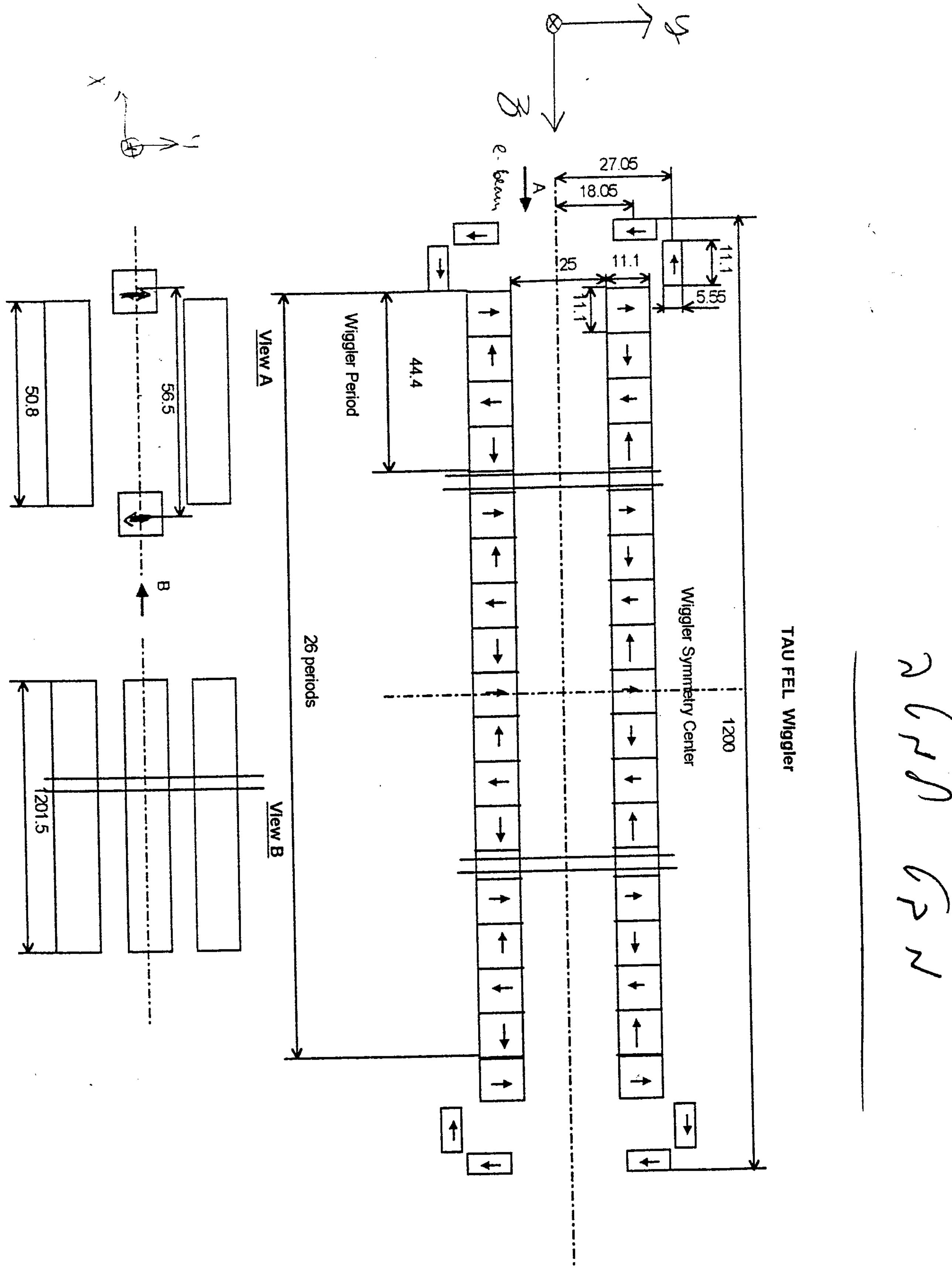


Fig. 2. TAU FEL Wiggler

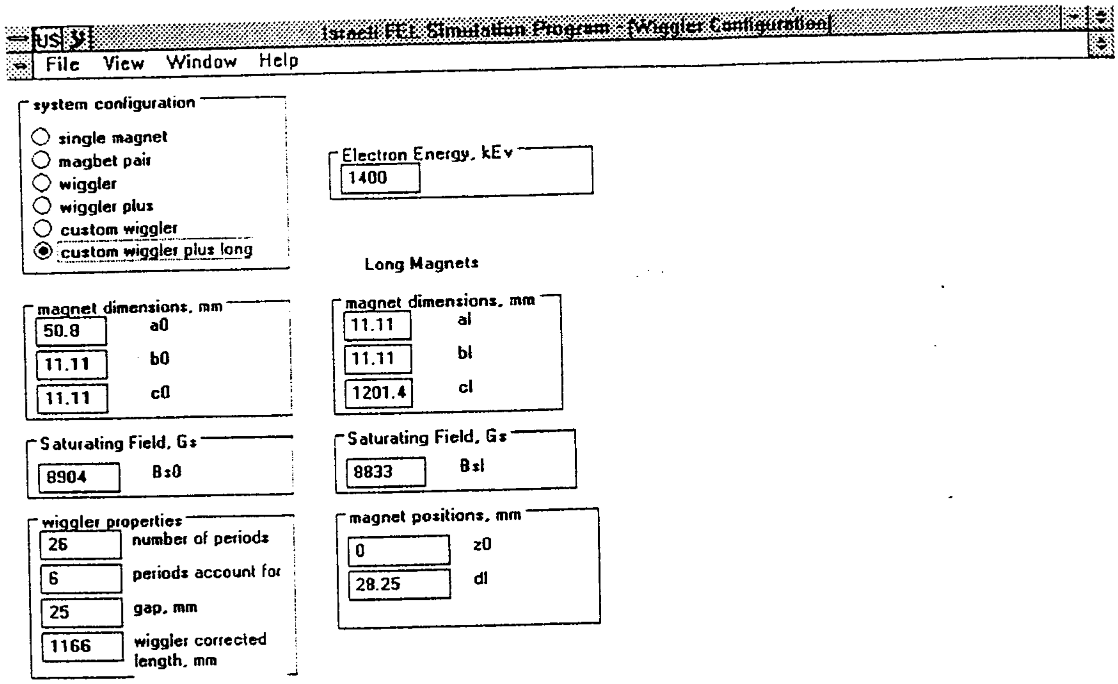


Table. 1. Wiggler configuration parameters .

Israeli FEL Simulation Program - [Additional Magnets]

File View Window Help

Entrance magnets: 2

To edit values double-click on the cell:

	a	b	c	d	l	alpha	Bs
1	50.8	11.11	5.55	18.05	19.94	180	9082
2	50.8	11.11	5.55	27.85	11.11	-90	9082

Exit magnets: 3

To edit values double-click on the cell:

	a	b	c	d	l	alpha	Bs
1	50.8	11.11	11.11	18.05	11.11	0	9082
2	50.8	11.11	5.55	27.05	22.22	90	9082
3	50.8	11.11	5.55	18.05	31.08	180	9082

מלבד זה ה wiggler
לאורך הוצאתו
בסוף ניסוי הלירה

Table 2. Additional magnets parameters.

Here are some explanations for the parameters in the tables.

<u>Table 1.</u>

a0,b0,c0 - dimensions of a single wiggler magnet in directions x,y,z, which are the local axes chosen for each magnet so that y-axis is parallel to the magnet magnetic field;

Bs0 - is the wiggler magnet saturating field ;

al,bl,cl - dimensions of the long magnets (measured in the same way as the wiggler magnets);

Bsl - long magnets saturating field ; I used here an "effective" value calculated in the following way:

$$\text{Bsl} \dashrightarrow \text{Bsl\_eff} = \text{Bsl} \cdot \frac{23 \cdot 50.8}{1201.5} = 0.97\ \text{Bsl}$$

This is because the long magnets are composed of 23 standard magnets (50.8 mm long) with some space between them to achieve a full length of 1201.5 mm.

z_0 - is the axial position of the center of the long magnet.

dl - is the lateral displacement of the long magnet (distance from the wiggler axis to the center of the long magnet);

number of periods: the number of full wiggler periods excluding the entrance and exit correction magnets;

periods account for: number of periods taken into account by the program for calclulation of the wiggler magnetic field at any point;

gap: face to face distance between the wiggler wings;

wiggler corrected length: the full wiggler length including first exit correction magnets pair, which turns the structure to be symetric (Note: this should be a dependent parameters $L' = b_0(4Nw + 1) = 1166.55$mm to be corrected in a future version).

Table 2:

a11,b11,c11,a12, ...,c23 - additional magnet dimensions; here first index (1 or 2) means entrance or exit magnet and second index indicates the order from left to right;

d11, ...d23, l11, ..., l23 - additional magnet positions; d is distance from wiggler axis to the center of magnet and l is distance along wiggler axis from center of additional magnet to center of 1-st wiggler magnet (in the entrance) or to center of the last wiggler magnet (in the exit).

alpha11, ... , alpha23 - angle of magnet direction. Zero angle corresponds to parallelizm with the 1-st wiggler magnet field direction, and positive angles are measured counter clockwise in the Y-Z plane (the rotation axis is the +X axis)..

Additional correction magnets purpose:

magnet 11 - angular shift correction;
magnet 12 - parallel displacement correction;
magnet 21 - symmetry correction full magnet;
magnet 22 - parallel displacement correction;
magnet 23 - angular shift correction;

Note: the wiggler, including entrance and exit additional magnets and the long magnets, is a completely symmetric structure.

Note: the Y axis of the right hand side cartezian coordinates system was chosen to correspond to a positive By value of the first magnet of the wiggler (without correcting magnets): By > 0.

3. Computer simulations.

(This section was not proof right)

We started with computer simulations with "ELOP" program to calculate optimal electron beam transport through wiggler.

The **first stage** of search for optimal electron beam transport was to find positions of the entrance correcting magnets which let an on-axis incoming electron propagat without betatron oscillation. These parameters are given in Table. 3

Israeli FEL Simulation Program - [Additional Magnets]

File View Window Help

Entrance magnets: 2

To edit values double-click on the cell:

	a	b	c	d	l	alpha	Bs
1	50.8	11.11	5.55	14.05	19.94	180	9082
2	50.8	11.11	5.55	31.05	11.11	-90	9082

Exit magnets: 3

To edit values double-click on the cell:

	a	b	c	d	l	alpha	Bs
1	50.8	11.11	11.11	18.05	11.11	0	9082
2	50.8	11.11	5.55	31.05	22.22	90	9082
3	50.8	11.11	5.55	14.05	31.08	180	9082

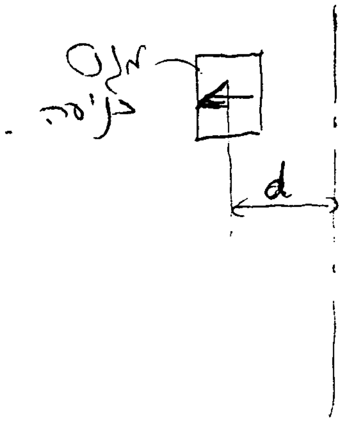


Table 3. Additional magnets parameters used for simulation.

Because of wiggler symmetry it is possible to consider only half of electron trajectory starting from the wiggler center with amplitude obtained in first stage.On the **second stage** we added to central electron 8 electrons so that they form ellipse in phase space (see A.Gover , "Basic Design Considerations for Free-Electron Lasers ..."). Parameters of this ellipse are chosen so :

$$\pi \cdot y_{b0} \cdot \varphi_{b0} = \varepsilon \qquad (8)$$

$$y_{b0} = \sqrt{\varepsilon / \pi k_b} \qquad (9)$$

This case is shown on Figs.3,4. Starting from wiggler center with "optimal" beam of 9 electrons we see how it exits wiggler. This gives us information about parameters of converging electron beam which is necessary to enter to wiggler (waist radius and position) to get optimal transport. These parameters are presented in Table 4. Electron beam optimal transport is presented in Figs. 5,6.

Parameters obtained in second stage we use in **next stage** as input for "QuadOpt" program, which allows to obtain optimal quads currents. Results are presented in Table 4.

emittance, mm*mrad	waist radius, mm	waist position, mm x	y	quads currents, A
3*π	0.36	12	19	0.94943 1.12929 1.38515 0.33832
10*π	0.67	11	23	1.31129 1.17783 1.32807 0.60644

Explanations to Table 4.
Effective waist position is position of waist of converging gaussian beam entering wiggler measured from wiggler front to the center of the wiggler.

Electron Beam Trajectories in FEL Wiggler

(ELOP simulatioin)

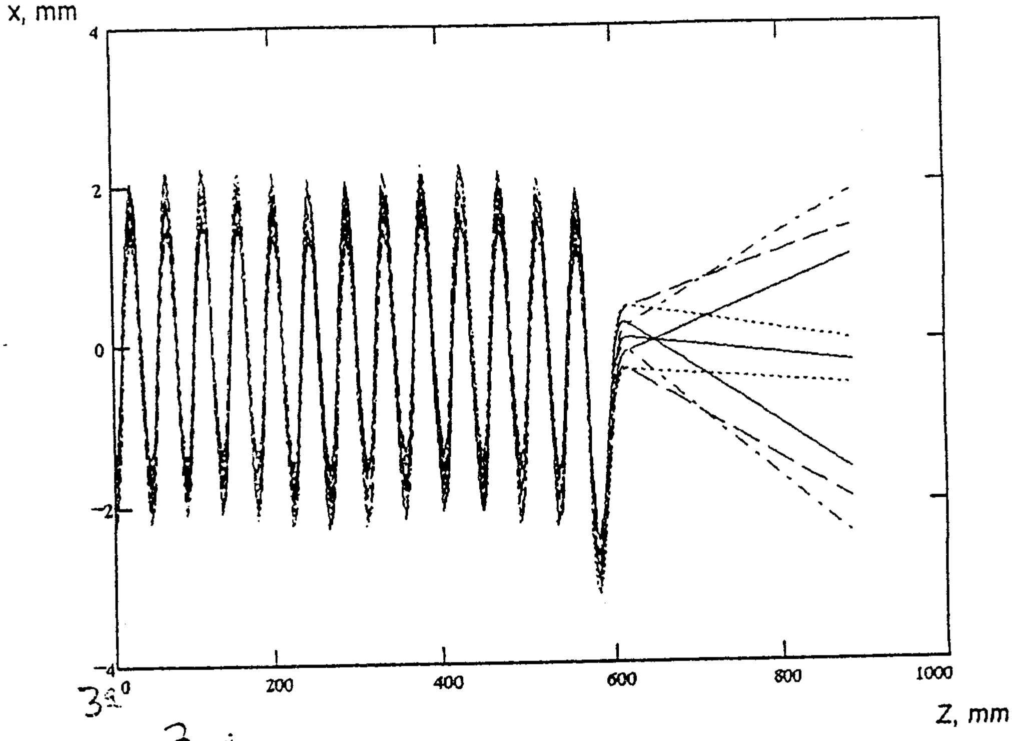
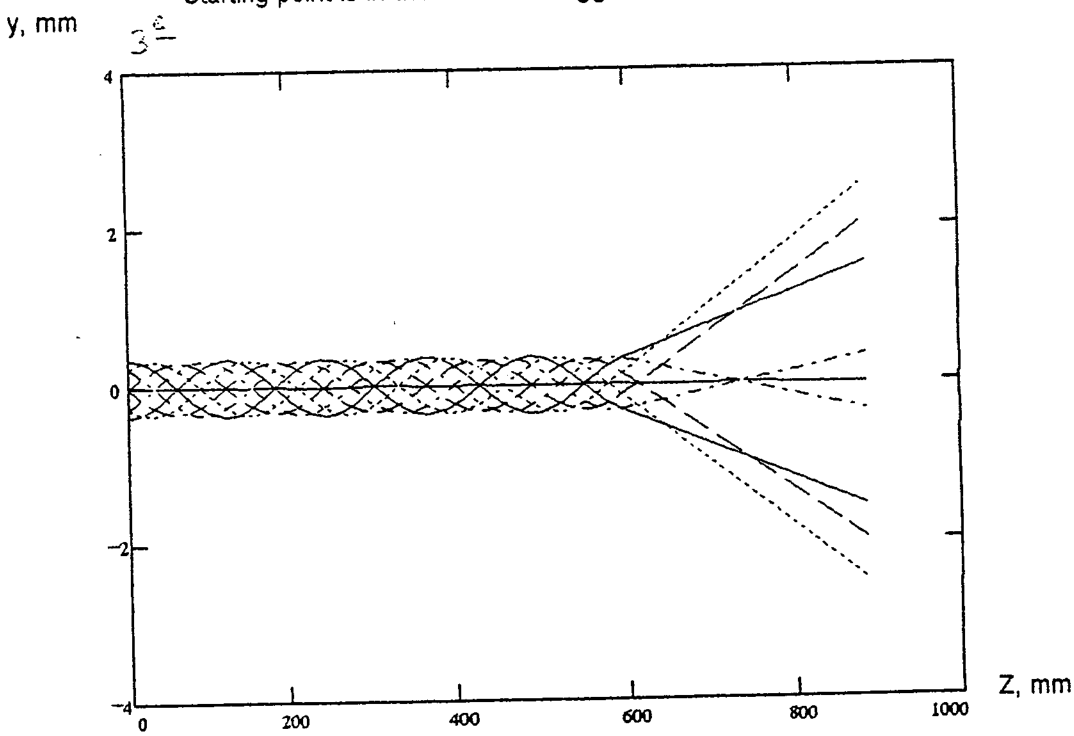


Fig. 3. 9-electron beam trajectories in wiggler.
Beam parameters: Rx=0.36 mm , αx=8.3 mrad, Ry=0.36 mm,
αy=8.3 mrad.emittance εx= εy= 3*π mm*mrad.
Starting point is in the center of wiggler.



Electron Beam Phase-Space Presentation

Z=0 mm

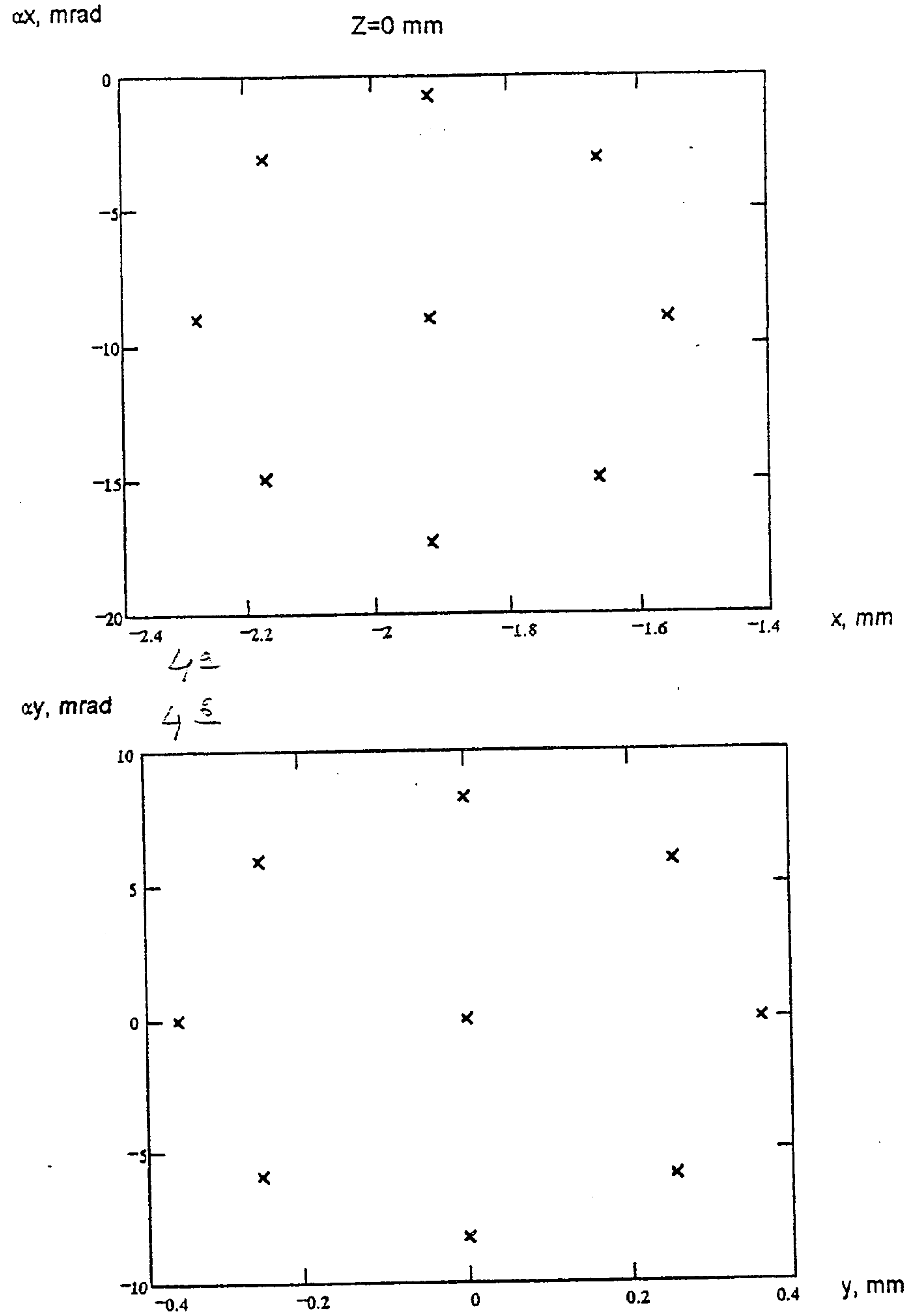
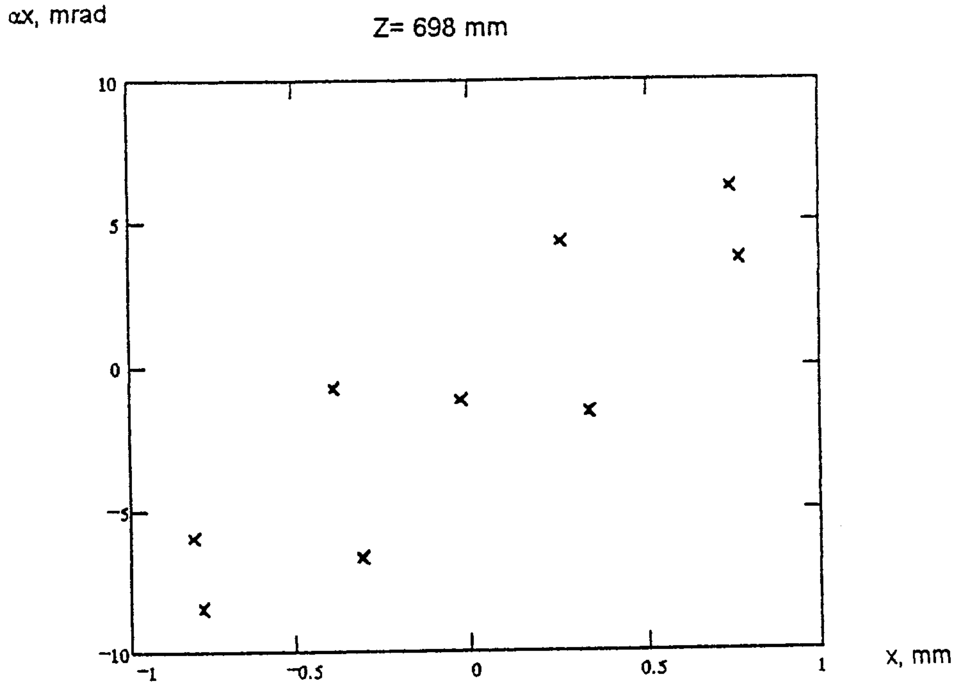


Fig. 4a-b

Electron Beam Phase-Space Presentation

Z= 698 mm



5 a

5 b

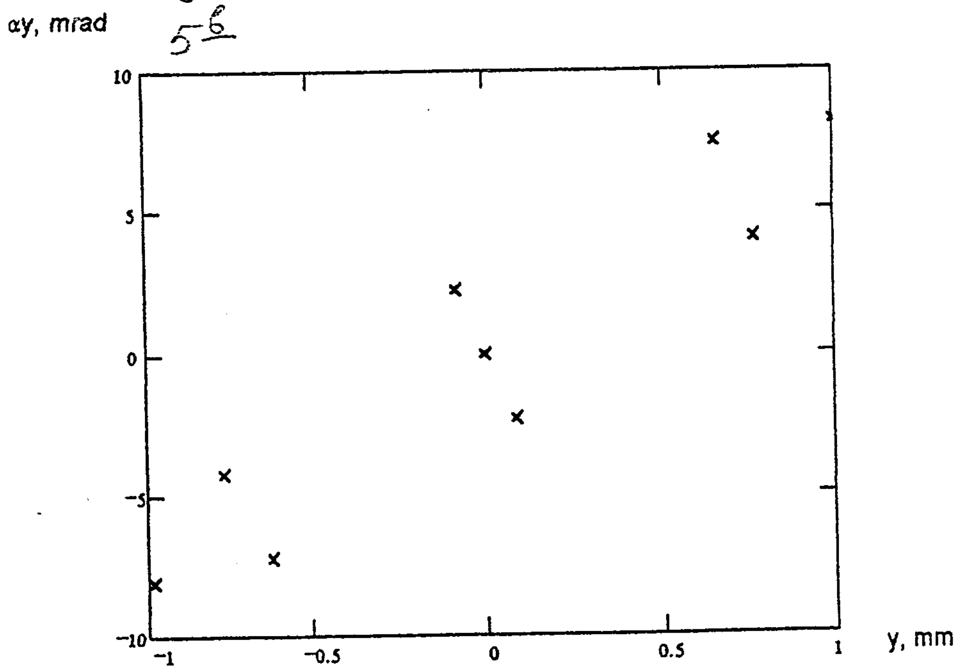


Fig. 5 a-b

Electron Beam Trajectories in FEL Wiggler

(ELOP simulatioin)

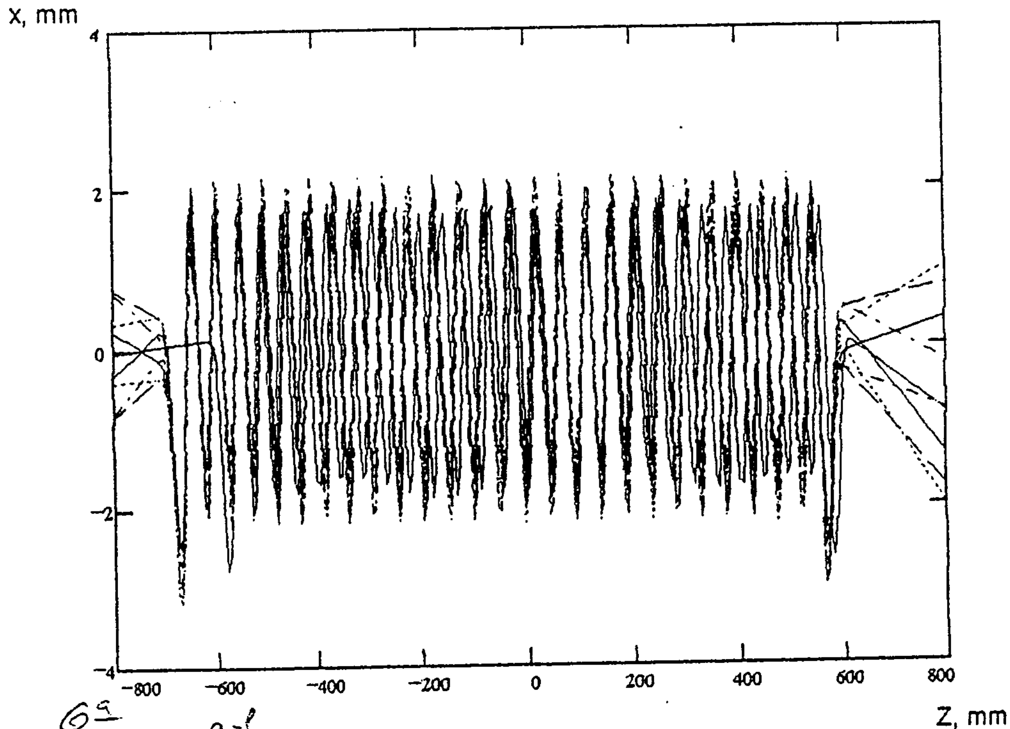


Fig. 12. 9-electron beam trajectories in wiggler .
Beam parameters: Rx=0.36 mm , αx=8.3 mrad, Ry=0.36 mm,
αy=8.3 mrad.emittance εx= εy= 3*π mm*mrad.
Starting point is in screen 2 (before wiggler)